BUCKLING NEAR A HOLE CENTERED IN A TENSION FIELD

JOHN F. DANIS

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John F. Danis

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AN INVESTIGATION OF THE BUCKLING CAUSED BY COMPRESSIVE FORCES EXISTING IN A THIN ELASTIC SHEET, WITH A HOLE IN THE CENTER, SUBJECT TO UNIFORM TENSION AT TWO EDGES

by

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

In a thin elastic sheet, with a hole in the center, subject to uniform tension on two opposite edges, a small area at the rim of the hole along the diameter of the hole parallel to the tension is in compression circumferentially.

This compression can cause buckling in this area. Such buckling may occur before the sheet yields at its point of maximum stress. Using Southwell's method of determining the critical buckling load, the relationship

$$\sigma_{cr} = KE \left(\frac{t}{d}\right)^2$$

is established, where σ_{cr} is the critical value of the uniaxial tension at a distance from the hole; d is the diameter of the hole; t is the thickness of the plate; E the (Young's) modulus of elasticity of the material; and K is an experimentally determined function of the ratio d/b of hole diameter to plate width.

This investigation was conducted at the United States Naval Postgraduate School, Monterey, California, in early 1957, by John F. Danis, Lieutenant, United States Navy, in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering.

The writer wishes to express his appreciation to Professors R. E. Newton and J. E. Brock, whose ideas, advice, and assistance made this paper possible.

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SYMBOLS

a -- Radius of Hole

a' -- Length of Sheet

b -- Width of Sheet

b' -- Constant Associated with Initial Curvature

d -- Diameter of Hole

h -- Thickness of Sheet (also designated t.)

k_y -- Expression for Curvature

l -- Length Measurement

t -- Thickness of Sheet

$w(x, y;P)$ - Primary Deflection due to Load

$w(x, y;0)$ - Initial Primary Deflection with no Load

x -- The Axis parallel to Applied Tension

y -- The Axis perpendicular to Applied Tension

A_i -- Coefficients which characterize Initial Deflection

B -- Strain gage reading for gages used to measure strain caused by change of radius of curvature at points of buckling.

B_o -- Initial Strain Gage Reading of B

B -- Difference in above. $(B - B_o)$

D -- Rigidity Function = $\frac{Eh^3}{12\ (1 - \sqrt{2})}$

D_1 -- Function of Initial Curvature = $B_o - b'$

E -- Modulus of Elasticity

I -- Moment of Inertia

EI -- Bending Modulus

K -- A function of d/b = $\frac{\sigma_{cr}}{E} \cdot \left(\frac{d}{t}\right)^2$

P -- Applied Load

P_i -- Critical Load for all Modes of Buckling

P_1 -- Critical Load for Primary Mode of Buckling

P_{cr} -- Critical Load as Determined by Experimentation

$W_n(x, y;P)$ -- Total Deflection due to all Modes of Buckling

$W_n(x, y;0)$ -- Initial Deflection due to all Modes of Buckling at No Load

β -- Modulus of Elastic Support

$\textcircled{\sim}$ -- Angle Measurement .

$\psi(x,y)$ -- Normal Function

τ -- Shearing Stress

σ -- Normal Stress

σ_{cr} -- Critical Buckling Stress = $\frac{Pcr}{bt}$

ν -- Poisson's Ratio

1. Introduction.

GENERAL DISCUSSION OF THE PHENOMENA AND THE EFFECTS

The problem of determining the stresses in an elastic plate under tension when the material is pierced by a circular hole is one of both theoretical interest and practical importance. Provided that the plate may be regarded as infinitely extended in two dimensions, the solution for a single hole is easily found and is well known. The presence of the hole leads to the occurrence of stresses equal to three times the tension at infinity, these maximum stresses occurring at the edge of the hole and on the diameter perpendicular to the direction of the applied tension. At the edge of the hole and on the diameter parallel to the direction of the applied tension a circumferential compressive stress occurs equal to the tension applied at infinity. For sheets of finite width, with a hole diameter equal to four-tenths of the width of the sheet, it has been found that this maximum stress is 3.74 times the applied stress, while the compressive stress has been found to be -1.44 times the applied stress. With a hole diameter equal to one-tenth of the width of the sheet, the maximum stress has been found to be 3.03 times the applied stress, while the compressive stress has been found to be -1.03 times the applied stress. (5)*

The presence of compressive stresses in a thin elastic sheet leads to the question of the condition under which buckling may occur. A search of the literature indicated that this question has not been investigated previously. (4) Accordingly, the investigation reported herein was undertaken for the purpose of experimentally determining the

* Numbers in parentheses refer to bibliographical references found on page 36.

conditions under which buckling would take place in such an elastic sheet pierced in the center by a circular hole and subjected to uniaxial tension.

In this analysis, it was assumed that Southwell's method of determining critical buckling load would be valid for all diameters of holes and all plate thicknesses. This assumption proved to be not altogether valid, but sufficient valid data were obtained to give legitimate results. It was necessary to obtain sufficient data on the buckling deflection to determine the critical buckling load, prior to yielding of the sheet at the point of maximum stress.

With this value of critical buckling load, the buckling stress was calculated; and, using the relationship,

$$\frac{\sigma_{cr}}{E} = K_{(d/b)} \left(\frac{t}{d}\right)^2$$

where $K_{(d/b)}$ is a function of hole diameter to plate width, K was evaluated for all sheets. This final result was plotted with K versus d/b.

2. Theoretical Analysis.

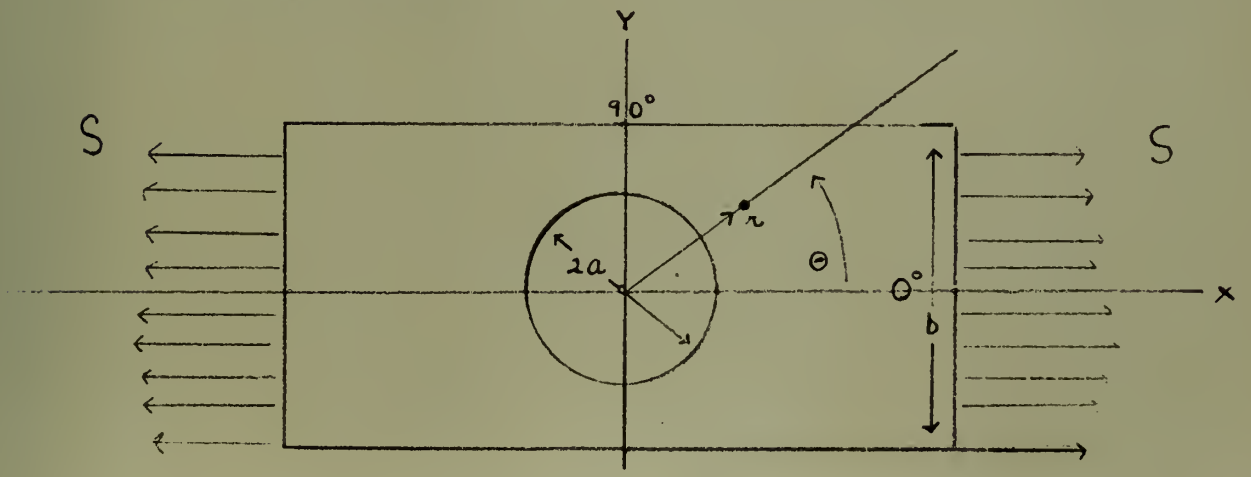

Figure 1